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ABSTRACT

A set of computer-oriented, supplemental activities is offered which can be used with a course in advanced algebra and trigonometry. The activities involve use of the BASIC programming language; it is assumed that the teacher is familiar with programming in BASIC. Students will learn some BASIC; however, the intent is not to develop proficient programmers. The idea is to allow students to observe results and to solve problems which usually are not accomplished because of time and difficulty. Nine units present student objectives and activities for the following topics: (1) getting acquainted with the computer (logging on and off, storing programs, learning the keyboard); (2) introduction to BASIC (writing programs); (3) functions and relations (linear functions); (4) systems of equations and inequalities; (5) polynomials; (6) sequences and series; (7) real numbers; (8) exponents and logs; and (9) trigonometry. This document is part of a collection of materials from the Iowa Area Education Agency 7 Teacher Center project. (DC)

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Advanced Algebra and Trigonometry

SUPPLEMENTAL COMPUTER UNITS

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AREA EDUCATION AGENCY 7

Cedar Falls, Iowa

Document title: Advanced Algegra and Trigonometry: Supplemental Computer Units

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Description:

A set of activities will be described which can be used with a course in advanced algebra and trigonometry.

These activities are for students who have access to a computer which will accept the BASIC langauge. It is assumed that the students have a good preparation in math including one year each of Algebra and Geometry.

The students will learn some BASIC as a result; however, the intent is not to teach students to become proficient programmers in BASIC. The idea is to allow students to observe results and to solve problems which are usually not accomplished because of time and difficulty. Hopefully, the activities will interest some students, so that they will investigate other topics independently.

General Course Objectives:

After completing the following activities during a course in Algebra II and Trigonometry, a student should be able to use a computer which accepts the BASIC language to investigate properties of real numbers and to solve problems. He should be aware of some ways in which computers can help him obtain results and insight into math topics, not necessarily be proficient in writing the programs in BASIC.

Activities Outline:

A. Getting Acquainted with Computer: log on, off, store programs, keyboard

B. Introduction to BASIC: writing programs operations, PRINT, LET, END, loops, IF THEN, GO TO (semicolons, commas and quotes)

C. Functions and Relations: Linear functions

D. Systems of Equations and Inequalities

E. Polynomials

F. Sequences and Series

G. Real numbers

H. Exponents and Logs

I. Trigonometry

UNIT A: Getting Acquainted with the Computer

Objectives:

1. To be able to log on and off the computer.

2. To write a simple program and run it.

3. To store a program and retrieve it successfully.

Activities:

1. "Log On" the computer and hand in evidence that you have done so.

2. Write a simple program which prints something or computes and RUN it. Hand in both the list of the program and the run. The first two statements in any program (in this course) will be: 10 REM (your name)
   20 REM (assignment).
   You may use more REM statements if you wish, but these two are mandatory!

3. Save the program you wrote; log off, then get it from library or disk. Hand in evidence of the procedure correctly completed.

UNIT B: Introduction to BASIC: Writing Programs

Objectives:

1. To write BASIC programs using the statements PRINT, LET, END, GO TO, IF, THEN, and REM.

2. To write an algebraic expression in BASIC and use the computer to evaluate it.

3. To write a BASIC program which will tabulate values of a function given its rule. (to use BASIC variables)

Activities:

1. Listen to lecture over elementary BASIC statements.

2. Write and RUN a BASIC program which will evaluate the following algebraic expressions:

   a. $\dfrac{(18.6)(45)^2 + 16}{1943 - 7}$

   b. $1/2 + 1/3 + 1/4 + 1/5 + 1/6$

   c. $\dfrac{2^8 - 1}{15}$

   d. $\sqrt{\dfrac{1^2 + 2^2 + 3^2 + 4^2 + 5^2}{1 + 2 + 3 + 4 + 5}}$

   Label the answers with the letters: a, b, c, d.

3. Write a program which will tabulate the values of the function $f(x) = 6x^3 - 5x^2 + 4x - 3$ for the domain $x \in \{-3, -2.9, -2.8, -2.7, \ldots, 0, \ldots 3\}$ in ordered pair notation.
   Plot the graph on a piece of co-ordinate paper using an appropriate scale.

UNIT C: Functions and Relations: Linear Functions

Objectives:

1. To use a computer to find members of a linear function.

2. To investigate properties of a linear function such as slope and graph.

3. To use the graphics or simulate graphics to plot the graph of a linear function.

Activities:

1. Write a BASIC program which will print out some solutions to the linear function $2x + 5y = 10$ in a table. Make a heading and have the equation printed as part of the title. (Use FOR-NEXT in your program.)

2. Listen to lecture or read about INPUT statement and the READ and DATA statements. Use either to write a BASIC program which will analyze a linear equation of the form $Ax + By = C$ and print out its

   a. slope
   b. x-intercept
   c. y-intercept

   with descriptions, of course. Allow for the possibility that either A or B might be zero.

3. Write a program which will print an equation of a line in $Ax + By = C$ form given co-ordinates of two points on the line. Allow for the possibility that the line may be vertical or horizontal.

4. Write a BASIC program which will graph the linear equation $y = 2x - 3$. Use either the available graphics on a computer such as the APPLE II or simulate graphics by plotting points using TAB( ) with the line printer. Choose your own scale.

   Optional: Graph the function $y = [x]$. Use the BASIC function INT(x).

5. Investigate $\frac{\Delta y}{\Delta x}$ which is defined as $\frac{y_1 - y_2}{x_1 - x_2}$ where (x_1, y_1) and (x_2, y_2) are ordered pairs of a given function.

Given the function, consider as its domain $\{-5 \leq x \leq 5; x \in I\}$ and write a program which will print out a table of values of the function, and of $\frac{\Delta y}{\Delta x}$.

Use these functions: $y = x^2 - x - 6$
Which is linear? $y = 3x_3 + 1$
Why? $y = 4x^3 - 4$
What about $\frac{\Delta y}{\Delta x}$? $y = 1 \times 1.$

Optional: Investigate $\frac{\Delta y}{\Delta x^2}$ which means $m_1 - m_2$ where m_1 and and m_2 are the respective slopes of 2 pairs of points on the graph.

UNIT D: Systems of Equations and Inequalities

Objectives:

1. To use the computer to find solutions to systems of linear equattions.

Activities:

1. Write a BASIC program which, given two linear equations, will determine the type of system, and print the unique solution if one exists.

2. Given 3 linear equations, write a BASIC program to print out the unique solution if one exists.

3. Write a BASIC program which will find the value of any 4th order determinant. Find the value of $\begin{vmatrix} 4 & -1 & 1 & 2 \\ 3 & -1 & 0 & 2 \\ 0 & 4 & 0 & 1 \\ 3 & 1 & 1 & 2 \end{vmatrix}$.

4. Linear Programming?

5. Write a BASIC program which uses Diophantine equations to find all positive integral solutions to the following equations:

   a. $2x + 5y = 12$
   b. $5x + 4y = 17$
   c. $3x + 7y = 20$
   d. $15x + 18y = 48$
   e. $22x + 7y = 87$

   Can you think of another method? If so, try it.

UNIT E: Polynomials

Objectives:

1. To determine how the BASIC language handles very large or very small numbers.

2. To use the computer to save time when working with multiplying or factoring polynomials.

Activities:

1. Write a BASIC program which will print out all the integral factors of a given integer.

2. Write a BASIC program which will determine the prime numbers from 2 to 99 and print them. Modify the program to find other primes larger than 100.

3. Write a BASIC program which will print out the coefficient and exponents of any given term of a binomial expansion such as $(2x - 3y^2)^8$.

4. Investigate how a computer handles very large or very small numbers. Show the results of computation or counting. How can you get more significant digits than the usual six?

UNIT F: Sequences and Series

Objectives:

1. To use the computer to generate sequences and partial sum of series.

2. To investigate convergence (and divergence) of sequences.

3. To use the computer to approximate irrational numbers such as $\sqrt{2}$.

Activities:

1. Write a BASIC program which will generate n terms of an arithmetic or geometric sequence and print them given the rule for the nth term.

2. Revise the program in #1 so that the sum of the first n terms is found and printed out. Do not use any formulas for the sum of a sequence.

3. Generate the first n Fibonacci numbers.

4. Write a BASIC program which will compute compound interest and the final amount for a given principal, rate, time, and number of interest periods per year. For example, find the value of $10,000 compounded quarterly at 9% for 5 years.

5. Write a BASIC program which will generate the terms of a convergent sequence for a given n.

a.	$a_n = 1/n$	1, 1/2, 1/3, 1/4, ...	Limit is 0
b.	$a_n = \frac{n+1}{n}$	2, 3/2, 4/3, ...	Limit is 1
c.	$a_n = 1 - (2/3)^n$	1/3, 5/9, 19/27, ...	Limit is 1
d.	$a_n = \frac{n}{2_n + 1}$	1/3, 2/5, 5/7, ...	Limit is 1/2

Write the program so that it will determine the smallest n which will guarantee the condition that the difference between a_n and the limit will be less than .001; than .0001.

6. Write a program which will generate at least 20 terms of the sequence of partial sums for each given series. Do they have a limit or not?

a. $1 + 1/2 + 1/4 + \ldots$ $\quad a_n = (1/2)^{n-1}$

b. $1 + 1/2 + 1/3 + \ldots$ $\quad a_n = 1/n$

c. $\frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \quad a_n = \frac{1}{n(n+1)}$

d. $2 - 6 + 18 - 54 + \ldots$

e. $1 - \frac{1}{4} + \frac{1}{4^2} - \frac{1}{4^3} + \ldots$

7. Write a BASIC program which will use the Binomial Theorem expansion to approximate $\sqrt{2}$ by writing it as $(1 + 1)^{\frac{1}{2}}$. Print the sequence of partial sums. How many terms are needed to be accurate to 6 decimal places? Can you do the same for $\sqrt{3}$?, $\sqrt{10}$?

UNIT G: Real Numbers

Objective:

1. To make use of the computer to investigate properties of rational numbers and real numbers without drudgeries of computation.

Activities:

1. Write a BASIC program which will print out all possible rational roots of a given polynomial equation. (See Activity 1; Unit E.)
   Then the program should determine which ones are actually roots. Test this on some of your homework problems.

2. Write a BASIC program which will print out, digit by digit, the decimal equivalent for any rational number less than 1. There are many possible methods for this program ranging from a simple approach to very sophisticated ideas. See what you can come up with. If it works, use it.

3. Write a BASIC program which will solve any quadratic formula in the form $ax^2 + bx + c = 0$.
   Your program should allow for three cases:

   a. no real roots
   b. 1 real root
   c. 2 real roots and print out accordingly.

4. Problems dealing with Golden Ratio:
   Given a rectangle with 1 as length and w as width; $\frac{1}{w} = \frac{1+w}{1}$ will be a property of a Golden Rectangle.
   Or considering a line segment AC where AB is 1 and BC is w:
   A ——— B ——— C . The line is divided at B in the golden ratio if the ratio of the larger part to the smaller equals the ratio of the whole to the larger.
   Find the positive numerical value for the ratio, $\frac{1}{w}$.
   Let us call this ratio, $\emptyset$, or phi. (Hint: let the width equal 1.) Find the negative solution. Call it $\emptyset'$.
   How are these two solutions related? What is their product?
   Verify that $\emptyset - 1 = \frac{1}{\emptyset}$.
   Write out a BASIC program which will print out the above results. You may use quadratic formula.

5. Consider the Fibonacci Sequence. (Unit F. Ex. #3) The sequence has many interesting properties. Investigate the related sequence whose terms are the ratios of any successive pair of Fibonacci numbers. (ie)

   1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, ... $\frac{a_n + 1}{a_n}$

   Investigate the sequence:

   1/1, 1/2, 2/3, 3/5, 5/8, ... $\frac{a_n}{a_n + 1}$

   Do these sequences converge? Is so, what is the limit?

6. Golden Series:

   1, Ø, 1 + Ø, 1 + 2Ø, 2 + 3Ø, 3 + 5Ø, ... additive

   1, Ø, $Ø^2$, $Ø^3$, $Ø^4$, ... geometric

   Write a BASIC program which generates the terms for each of these series. How do they compare? Why?

7. Write a BASIC program which will simulate the synthetic division process to help you find the rational roots of a polynomial equation. You should input the degree and coefficients of the equation to start.

8. Write a BASIC program which will allow you to analyze a function, finding integral roots, approximate maximums and minimums, and increasing or decreasing intervals. Use the statement to define a special function:

   10 DEF FNA(x) = x ↑ 2 + 8, for example.

9. Write a BASIC program to approximate a zero of a function once an interval in which it occurs is found.

10. Write a BASIC program which will, given the formula for a quadratic function $y = ax^2 + bx + c$:

    a. print out the co-ordinates of the vertex.
    b. tell whether that point is a max or min.
    c. print out some points on the graph of the curve.

UNIT H: Exponents and Logarithms

Objectives:

1. To use the computer to evaluate expressions involving very large or small exponents.

2. To investigate the definition of logarithms using the computer.

3. To use the computer with the compound interest formula.

4. To investigate series involving the irrational number e.

Activities:

1. Write a BASIC program which will evaluate numerical expressions involving radicals and exponents and logarithms. Check your homework with this.

2. Write a BASIC program which will compare the value of money invested, $10,000, at 8% of 10 years, when the number of interest periods per year varies from 1, 2, 4, 12, 360 (daily). Is there a significant difference in the value of the investment?

3. The base for natural logarithms, e, can be determined to any number of decimal places by evaluating the series if you make n large enough, $n \in I+$.

$$e = 1 + \frac{1}{1} + \frac{1}{1.2} + \frac{1}{1.2.3} + \ldots + \frac{1}{(n-1)}\ !$$

Write a program to find an approximation for e. Stop when you find a term less than 5×10^{-6}. Print each term and each sum as they accumulate.

4. It can be shown by advanced methods that

$$\log_e(1 + x) = x - \frac{1}{2}x^2 + \frac{1}{3}x^3 - \frac{1}{4}x^4 + \ldots$$

$$\text{for } -1 < x \leq 1$$

$$\text{where } a_n = \frac{(-1)^{n+1}x^n}{n}\ .$$

Write a BASIC program to find $\log_e(1 + x)$ when you input x. Print each term and the accumulated sum. Stop when you reach a term which is less than 5×10^{-8}. Compare the final sum by printing out the value of log (1 + x) using the BASIC function LOG. Try $x \in \{0.1, 0.3, 0.5\}$.

UNIT I: Trigonometry

Objectives:

1. To use the computer to graph trigonometric functions.

2. To solve triangles using trigonometric methods

Activities:

1. Consider the set of degree measures: {10, 20, 30, ... 360}. Print out a table of degree and their corresponding radian measures.

2. Write a BASIC program which will make a table with the heading | θ | sinθ | cosθ | tanθ | for the special angles; multiples of $\pi/4$ and $\pi/3$, from 0 to 2π radians.

3. Write a BASIC program which uses the available graphics on a micro-computer to generate a graph of the functions

   y = sinx.
   y = cosx.
   y = sin2x.
   y = cos2x.
   y = 2 sinx.
   y = 2 cosx. etc.
   y = sin(x + π).
   y = sin(x + $\pi/2$).

4. Write a BASIC program which will solve any right triangle given

   a. angle and adjacent side
   b. angle and opposite side
   c. angle and hypotenuse
   d. adjacent side and hypotenuse
   e. adjacent side and opp. side
   f. opp. side and hypotenuse

5. Write a BASIC program which will solve any triangle given

   a. angle, side, angle
   b. side, angle, side
   or c. side, side, and angle